MATHEMATICS SEMINAR SOLUTIONS

1 a) $y + \frac{1}{y} = 2.5$

$y^2 - 2.5y + 1 = 0$

$2y^2 - 5y + 2 = 0$

$2y^2 - 4y - y + 2 = 0$

$(y - 2)(2y - 1) = 0$

$y = 2 = \log_x 16 \; or \; y = \frac{1}{2} = \log_x 16$

$\sqrt{x} = 16 \; or \; x^2 = 16$

$x = 256 \; or \; x = 4 \; (discard \; x = -4)$

b) $2^p + 2^q = 320$ and $p + q = 14$

$2^{p+q} = 2^p \times 2^q = 2^{14} = 16384.$ Let $2^p = x \; and \; 2^q = y$

$x + y = 320 \ldots\ldots (i) \qquad xy = 16384 \ldots\ldots\ldots (ii)$

$x(320 - x) = 16384$

$320x - x^2 = 16384, thus \; x^2 - 320x + 16384 = 0$

$$x = \frac{320 \pm \sqrt{(-320)^2 - 4 \times 1 \times 16384}}{2 \times 1}$$

$x = 256 = 2^p \; or \; x = 64 = 2^p$

$2^p = 256 \; for \; 2^q = 64$ or $2^p = 64 \; for \; 2^q = 256$

$2^p = 2^8 \; for \; 2^q = 2^6$ or $2^p = 2^6 \; or \; 2^q = 2^8$

$p = 8 \; for \; q = 6 \; or \; p = 6 \; for \; q = 8.$

2 a) $(x + iy)(x + iy)^2 = x^2 - y^2 + 2ixy$

$$x^2 - y^2 + 2ixy = 7 - 6\sqrt{2}i$$

Equating real and complex parts.

$$x^2 - y^2 = 7 \; or \; 2xy = -6\sqrt{2}$$

On solving

$x = 3 \; when \; y = -\sqrt{2} \; or \; x = -3 \; when \; y = \sqrt{2}$

b) (i) $f(z) = 2z^3 - 4z^2 - 5z - 3$

$f(3) = 2x0^3 - 4x0^2 - 5x0 - 3$

$f(3) = 0$

(ii) $2z^3 - 4z^2 - 5z - 3 = (z-3)(2z^2 + 2z + 1)$

On solving

$$\text{z} = 3\,, \text{z} = \frac{-1}{2} + \frac{\text{i}}{2}\,, \text{z} = \frac{-1}{2} - \frac{\text{i}}{2}$$

3 a) $Let \; n = number \; of \; three - moonth \; periods$

$S_n = 3000000, P = 500000, R = 3, time \; = n \; three - months \; periods$

$Let \; A_k = P\left(1 + \frac{R}{100}\right)^{n+1-k}$ be amount got after the k^{th} deposit made

$A_1 = 50000\left(1 + \frac{3}{100}\right)^n = 50000(1.03)^n$

$A_2 = 50000(1.03)^{n-1}$

$A_3 = 50000(1.03)^{n-2}$

$A_n = 50000(1.03)^1$

Total $= S_n = A_n + \cdots + A_3 + A_2 + A_1$

$3000000 = 50000[(1.03)^1 + (1.03)^2 + \cdots + (1.03)^{n-2} + (1.03)^{n-1} + (1.03)^n]$

$3000000 = 50000[1.03(\frac{1.03^n - 1}{1.03 - 1})]$

$1.03^n = \frac{3000000 \times 0.03}{50000 \times 1.03} + 1$

$1.03^n = 2.747573$

$n = \frac{Ln\, 2.747573}{Ln\, 1.03}$

$n = 34.1935$

Number of months $= 3 \times 34.1935 = 102.58 \; (4 \; s.f)$

b) $\sqrt[3]{\left(\frac{1-x}{1+x}\right)}$

$$= (1-x)^{\frac{1}{3}}(1+x)^{-\frac{1}{3}} \text{ for } n = \frac{1}{3}, x \equiv (-x) \text{ and } n = -\frac{1}{3}, x \equiv x$$

$$(1-x)^{\frac{1}{3}} = 1 - \frac{1}{3}x - \frac{2}{9}x^2\left(\frac{1}{2}\right) + \cdots$$

$$(1+x)^{-\frac{1}{3}} = 1 - \frac{1}{3}x + \frac{4}{9}x^2\left(\frac{1}{2}\right) + \cdots$$

$$\sqrt[3]{\left(\frac{1-x}{1+x}\right)} = \left[1 - \frac{1}{3}x - \frac{1}{9}x^2 + \cdots\right]\left[1 - \frac{1}{3}x + \frac{2}{9}x^2 + \cdots\right]$$

$$\sqrt[3]{\left(\frac{1-x}{1+x}\right)} = 1 - \frac{1}{3}x + \frac{2}{9}x^2 - \frac{1}{3}x + \frac{1}{9}x^2 - \frac{1}{9}x^2 + \cdots$$

$$\sqrt[3]{\left(\frac{1-x}{1+x}\right)} = 1 - \frac{2}{3}x + \frac{2}{9}x^2$$

4. a) $f(n) = 3^{2n} + 5^n - 2$

$For\ n = 1, f(1) = 3^2 + 5^1 - 2 = 12$

$f(1) = 4 \times 3$. Hence true for n=1.

$For\ n = k > 1, assume\ f(k) = 3^{2k} + 5^k - 2 = 4A\ is\ a\ multiple\ of\ 4$

$\therefore 3^{2k} = 4A - 5^k + 2$

$For\ n = k+1, f(k+1) = 3^{2(k+1)} + 5^{k+1} - 2$

$f(k+1) = 9(3^{2k}) + 5(5^k) - 2 = 9[4A - 5^k + 2] + 5(5^k) - 2$

$f(k+1) = 36A + 18 - 9(5^k) + 5(5^k) - 2$

$f(k+1) = 36A + 16 - 4(5^k) = 4[9A + 4 - 5^k]$

Since $k \ll A > 3, then\ 9A + 4 - 5^k\ is\ a\ positive\ integer.$

$f(k+1)\ is\ divisible\ by\ 4. True, 3^{2n} + 5^n - 2$

$is\ a\ multiple\ of\ 4\ for\ all\ integers, n \geq 1$

b) $\sum_{r=1}^{2n} r^3 = \frac{1}{4}(2n)^2(2n+1)^2$

$= n^2(2n+1)^2$

$\sum_{r=1}^{2n} r^3 = 1^3 + 2^3 + 3^3 \dots + n^3 + (n+1)^3 + \cdots + (2n-1)^3 + (2n)^3$

$\sum_{r=1}^{2n} r^3 = 1^3 + 3^3 + 5^3 \dots + (2n-1)^3 + 2^3[1^3 + 2^3 + 3^3 \dots + n^3]$

$n^2(2n+1)^2 = 1^3 + 3^3 + 5^3 \dots + (2n-1)^3 + 8[\frac{1}{4}(n)^2(n+1)^2]$

$1^3 + 3^3 + 5^3 \dots + (2n-1)^3 = n^2(2n+1)^2 - 2n^2(n+1)^2$

$= n^2[(2n+1)^2 - 2(n+1)^2$

$= n^2[4n^2 + 4n + 1 - 2n^2 - 4n - 2]$

$\sum_{r=1}^{n}(2r-1)^3 = n^2(2n^2 - 1)$

5.a) For x-2

$P(2) = 2^3 + a2^2 + b$

$P(2) = 8 + 4a + b$

P(2)= 0

$4a + b = -8 - - - - - - - - - - - (i)$

$Q(-1) = -1 + a + b$

$a + b = -14 - - - - - - - - - - - (ii)$

On solving the two equations

$a = 2 \ and \ \ b = -16$

b) The least value is -18

6. a) $\frac{5}{x+1} + \frac{1}{x-2} - \ \frac{3}{(x-2)^2}$

b) $\frac{15}{4} - 6x + \frac{69}{16}x^2$

7.a) a) $\sqrt{x+7} = 2 + \sqrt{x-1}$

$x + 7 = 4 + 4\sqrt{x-1} + x - 1$

$1 = \sqrt{x-1}$

$1 = x - 1$

$x = 2$

Checking: $L.H.S = \sqrt{2+7}\ = 3$

$R.H.S = 2 + \sqrt{2-1}\ = 3$

$L.H.S = R.H.S, hence\ x = 2$

b) $\frac{Z+1}{Z-i}\ = \frac{x+iy+1}{x+iy-i}$

$= \frac{(x+1)+iy}{x+i(y-1)}$

$= \frac{[(x+1)+iy][x-i(y-1)]}{[x+i(y-1)][x-i(y-1)]}$

$= \frac{x(x+1)+y(y-1)+i[xy-(x+1)(y-1)]}{x^2+(y-1)^2}$

$= \frac{x(x+1)+y(y-1)}{x^2+(y-1)^2} + \frac{[xy-(x+1)(y-1)]}{x^2+(y-1)^2}i$

$= \frac{x^2+x+y^2-y}{x^2+(y-1)^2} + \frac{-y+x+1}{x^2+(y-1)^2}i, for\ a = \frac{x^2+x+y^2-y}{x^2+(y-1)^2}, b = \frac{-y+x+1}{x^2+(y-1)^2}.$

Hence, $\tan^{-1}\frac{a}{b} = \tan^{-1}\left[\frac{-y+x+1}{x^2+(y-1)^2} \div \frac{x^2+x+y^2-y}{x^2+(y-1)^2}\right] = -\frac{\pi}{4}$

$\frac{-y+x+1}{x^2+x+y^2-y} = -1$

$x^2 + x + y^2 - y = y - x - 1$

$x^2 + 2x + y^2 - 2y + 1 = 0\ is\ circle\ x^2 + y^2 + 2gx + 2fy + c = 0$

for constants f, g, and c.

8.a) $\frac{n!}{(n-3)!3!} + \frac{n!}{(n-2)!2!} = 4n$

$\frac{n(n-1)(n-2)}{6} + \frac{n(n-1)}{2} = 4n$

$\frac{1}{6}(n^3 - n) = 4n$

$n(n^2 - 25) = 0$

$n = 5$

b) Introducing In on both sides

$$Ine^{3x+4y} = In2e^{2x-y}$$

$$3x + 4y = In\ 2 + 2x - y$$

$$x + 5y = In\ 2 \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

$$x - 5y = In\ 8 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Solving the two equations simultaneously

$$x = 2In\ 2\ \ and\ y = -\frac{1}{5}In\ 2$$

c) (i)

$For\ 2x + 9 \geq 0$

$-x - 3 < 2x + 9$ $ALT: (-x - 3)^2 = (2x + 9)^2$ etc

$-3x < 12$

$\frac{-3x}{-3} > \frac{12}{-3}$

$x > -4$

$For\ (2x + 9) < 0, |2x + 9| = -(2x + 9)$

$-x - 3 > -(2x + 9)$

$x + 3 > 2x + 9$

$-x > 6$

$\frac{-x}{-1} < \frac{6}{-1}$

$x < -6$

c) (ii) $x = 0\ , x = 1\ , x = 2$

9 (a) $p = 3\ or\ q = -10$

(b) $\alpha + \beta = -\frac{b}{a}$

$\alpha\beta = \frac{c}{a}$

Sum of roots = $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$$\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c}{a}}$$

$$\frac{b^2 - 2ac}{ac}$$

Product =1

$$acx^2 - (b^2 - 2ac)x + 1 = 0$$

(c) $r = 0.6\ and\ a = 2$

10.a) $3\,\mathrm{Sin}\,A\cos\alpha - 3\cos A\sin\alpha = \cos A\cos\alpha - \sin A\sin\alpha$

$3\,\mathrm{Sin}\,A\cos\alpha + \sin A\sin\alpha = 3\cos A\sin\alpha + \cos A\cos\alpha$

$\mathrm{Sin\ A}\ (3\cos\alpha + \sin\alpha) = \cos A(3\sin\alpha + \cos\alpha)$

$\frac{3\mathrm{Cos}\,\alpha+\sin\alpha}{3\,\mathrm{Sin}\,\alpha+\cos\alpha} = \frac{\cos A}{\sin A}$

$\mathrm{Cot}\,A = \frac{(\,3\cos\alpha+\sin\alpha)/sin\alpha}{(3\sin\alpha+\cos\alpha)/sin\alpha}$

$\mathrm{Cot}\,A = \frac{3\cot\alpha+1}{3+\cot\alpha}$

$Hence\ tan\ A = \frac{3+\cot\alpha}{3\cot\alpha+1} = \frac{3+(-\frac{1}{2})}{3(-\frac{1}{2})+1}$

$tan\ A = -5\ \ with\ tan\,\alpha = -2.$

$tan\ (A+\alpha) = \frac{\tan A+\tan\alpha}{1-\tan A\tan\alpha}$

$tan\ (A+\alpha) = \frac{-5+\ -2}{1-(-5\times-2)}$

$tan\ (A+\alpha) = \frac{7}{9}$

$b)\ cos\,x = \sin\frac{x}{2}$

$1 - 2sin^2\frac{x}{2} = \sin\frac{x}{2}$

$2sin^2\frac{x}{2} + \sin\frac{x}{2} - 1 = 0$

$\sin\frac{x}{2} = \frac{-1\pm\sqrt{(1)^2-4\times1\times-1}}{2\times2}$

$\sin\frac{x}{2} = \frac{1}{2}$ but $\sin\frac{x}{2} \neq -1$ for $0^o \leq \frac{x}{2} \leq 90^o$

$\frac{x}{2} = \sin^{-1}\frac{1}{2} = 30^o$

11.a) $\frac{2tan2A}{1+tan^2 2A} = \frac{2tanA}{1+tan^2 A}$ ALT 1: $2cos3AsinA = 0$ etc

$\frac{2(\frac{2t}{1-t^2})}{1+(\frac{2t}{1-t^2})^2} = \frac{2t}{1-t^2}$ ALT 2: $2sin2Acos2A - sin2A = 0$ etc

$\frac{4t(1-t^2)}{(1-t^2)^2+4t^2} = \frac{2t}{1-t^2}$

$\frac{2t(1-t^2)}{(1+t^2)^2} = \frac{t}{1+t^2}$

$2t(1-t^2)(1+t^2) - t(1+t^2)^2 = 0$

$1+t^2 \neq 0 \implies t(1+t^2)[2(1-t^2) - (1+t^2)] = 0$

$t(2-2t^2-1-t^2) = 0$

$t(1-3t^2) = 0$

$t = 0$ or $t = \pm\frac{\sqrt{3}}{3}$

$A = \tan^{-1} 0,\ \tan^{-1}(-\frac{\sqrt{3}}{3})$ or $\tan^{-1}(\frac{\sqrt{3}}{3})$

$A = 180^o, 150^o, 330^o, 30^o or\ 210^o$

$\frac{2t}{1-t^2} = tan2A$ is undefined if $1 - t^2 = 0, for\ t = \pm 1$ for $A = 90^o$ or 270^o

$A = 30^o, 90^o, 150^o, 180^o, 210^o, 270^o, and\ 330^o$.

b) $L.H.S = \frac{(sin\alpha+cos\beta)^2-(cos\alpha-sin\beta)^2}{2(sin\alpha+cos\beta)(cos\alpha-sin\beta)}$

$= \frac{sin^2\alpha+cos^2\beta+2\sin\alpha\cos\beta-cos^2\alpha-sin^2\beta+2\cos\alpha\sin\beta}{2\sin\alpha\cos\alpha+2\cos\alpha\cos\beta-2\sin\alpha\sin\beta-2cos\beta\cos\beta}$

$= \frac{cos^2\beta-sin^2\beta-(cos^2\alpha-sin^2\alpha)+2(\sin\alpha\cos\beta+\cos\alpha\sin\beta)}{2\sin\alpha\cos\alpha-2\sin\beta\cos\beta+2(\cos\alpha\cos\beta-\sin\alpha\sin\beta)}$

$= \frac{\cos 2\beta-\cos 2\alpha+2\sin(\alpha+\beta)}{\sin 2\alpha-\sin 2\beta+2\cos(\alpha+\beta)}$

$\frac{-2\sin(\beta+\alpha)\sin(\alpha-\beta)+2\sin(\alpha+\beta)}{2\cos(\beta+\alpha)sin(\beta-\alpha)+2\cos(\beta+\alpha)}$

$= \frac{\sin(\alpha+\beta)[\sin(\alpha-\beta)+1]}{\cos(\alpha+\beta)[\sin(\alpha-\beta)+1]}$

$= \tan(\alpha+\beta) = R.H.S$

12.a) $R.H.S = \frac{a-b}{a+b}\cot\frac{C}{2}$

$= \frac{2R(\sin A - \sin B)}{2R(\sin A + \sin B)}\cot\frac{C}{2}$

$= \frac{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}\cot\frac{C}{2}$

$= \frac{\cos(90^o - \frac{C}{2})}{\sin(90^o - \frac{C}{2})}\tan\frac{A-B}{2}\cot\frac{C}{2}$

$= \frac{\text{Sin}\frac{C}{2}}{Cos\frac{C}{2}}\tan\frac{A-B}{2} \times \frac{cos\frac{C}{2}}{\text{Sin}\frac{C}{2}}$

$= \tan\frac{A-B}{2} = L.H.S$

b) $a = 4\ cm, b = 6cm$ and $C = 127.2^o$

$\tan\frac{A-B}{2} = \frac{4-6}{4+6}\cot\frac{127.2^o}{2}$

$\tan\frac{A-B}{2} = -0.099281$

$\frac{A-B}{2} = -5.6698^o \ldots\ldots\ldots\ldots\ldots (i)$

$\frac{A+B}{2} = \frac{180^o - 127.2^o}{2} = 26.4^o \ldots\ldots.(ii)$

$(i) + (ii),\ A = 20.73^o$

$(ii) - (i),\ B = 32.07^o$

13.a) $tan \propto = \frac{6}{4}$

$\propto = 56.31°\ \ , R = 2\sqrt{13}$

$$7.21\sin(\theta - 56.31°)$$

b i)

$$7.21\sin(\theta - 56.31°) = 0$$

$$\theta = 80.9°$$

b ii) Maximum = 49

Minimum = - 3

14.a) a) $x = 3 + 2\alpha - \beta \ldots \ldots (i)$

ALT: $\boldsymbol{n} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \times \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$

$y = 1 - 3\alpha + 2\beta \ldots \ldots (ii)$

$= \begin{pmatrix} -3 \times 3 - 1 \times 2 \\ 1 \times -1 - 2 \times 3 \\ 2 \times 2 - 3 \times -3 \end{pmatrix} = \begin{pmatrix} -11 \\ -7 \\ 1 \end{pmatrix}$ etc

$z = -2 + \alpha + 3\beta \ldots \ldots (iii)$

$2(i) + (ii) \implies 2x + y = 7 + \alpha$

$3(i) + (iii) \implies 3x + z = 7 + 7\alpha$

$\implies 3x + z = 7 + 7(2x + y - 7)$

$11x + 7y - z - 42 = 0$

b) $\boldsymbol{d} = \begin{pmatrix} 11 \\ 7 \\ -1 \end{pmatrix}$ is parallel to the line

The $\perp$ $line\ is$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ -7 \\ 25 \end{pmatrix} + t\begin{pmatrix} 11 \\ 7 \\ -1 \end{pmatrix}$

$x = -5 + 11t,\ y = -7 + 7t,\quad z = 25 - t \quad \ldots \ldots (i)$

$11(-5 + 11t) + 7(-7 + 7t) - (25 - t) = 42$

$-55 + 121t - 49 + 49t - 25 + t = 42$

$171t = 171$

$t = 1$

$x = -5 + 11, y = -7 + 7, z = 25 - 1$

$x = 5, y = 0, z = 24$

The foot is (6,0,24)

15.a) $\boldsymbol{d} = \begin{pmatrix} 7 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} -1 \\ -6 \\ 6 \end{pmatrix} = \begin{pmatrix} 8 \\ 10 \\ -4 \end{pmatrix}$ *is direction vector*

$\emptyset$ *is* $\boldsymbol{r} = (7 + 8\mu)\boldsymbol{i} + (4 + 10\mu)\boldsymbol{j} + (2 - 4\mu)\boldsymbol{k}$

b) $\boldsymbol{d_1}.\boldsymbol{d_2} = \begin{pmatrix} 8 \\ 10 \\ -4 \end{pmatrix}.\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$

$= 24 - 20 - 4$

$\boldsymbol{d_1}.\boldsymbol{d_2} = 0,$ *hence perpendicular*

Also, $3\lambda = 7 + 8\mu \ldots.(i)$

$1 - 2\lambda = 4 + 10\mu \ldots.(ii)$

$3 + \lambda = 2 - 4\mu \ldots.(iii)$

(i) + 2(iii) $3\lambda + 6 + 2\lambda = 7 + 4$ hence $5\lambda = 5$

$\lambda = 1$

$\mu = \frac{1}{8}(3 - 7) = -\frac{1}{2}$

$y = 4 + 10\left(-\frac{1}{2}\right) = -1$

Also, $y = 1 - 2(1) = -1$

c) $\boldsymbol{d_3} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 8 \\ 10 \\ -4 \end{pmatrix}$

$= \begin{pmatrix} -2 \times -4 \times 10 \\ 1 \times 8 - 3 \times -4 \\ 3 \times 10 - 8 \times -2 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ 20 \\ 46 \end{pmatrix}$

Line is $\begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} + t\begin{pmatrix} -2 \\ 20 \\ 46 \end{pmatrix}$

16.a) . $\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ -1 \\ -3 \end{pmatrix} = \left|\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}\right| \times \left|\begin{pmatrix} -2 \\ -1 \\ -3 \end{pmatrix}\right| \cos\theta$ ALT: $\left|\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ -1 \\ -3 \end{pmatrix}\right| = \left|\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}\right| \times \left|\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}\right| \sin\alpha$

etc

$-4 - 3 - 3 = \sqrt{(4 + 9 + 1)} \times \sqrt{(4 + 1 + 9)} \cos\theta$
$-10 = 14\cos\theta$

$$\cos\theta = -\frac{5}{7}$$

$$\theta = \cos^{-1}(-\frac{5}{7})$$

$$\theta = 135.58^0$$

$$Required, \alpha = 135.58^0 - 90^0$$

$$= 45.58^0$$

b) $r = |\boldsymbol{PQ}| = \sqrt{\{(2+3)^2 + (-1-2)^2 + (3-1)^2\}}$ $ALT: \overrightarrow{PQ} = \begin{pmatrix} -5 \\ 3 \\ -2 \end{pmatrix}$

$q = |\boldsymbol{PR}| = \sqrt{\{(2-1)^2 + (-1-3)^2 + (3+2)^2\}}$ $\overrightarrow{PR} = \begin{pmatrix} -1 \\ 4 \\ -5 \end{pmatrix}$. Get normal etc

$p = |\boldsymbol{QR}| = \sqrt{\{(-3-1)^2 + (2-3)^2 + (1+2)^2\}}$

$r = \sqrt{38}$, $q = \sqrt{42}$, $p = \sqrt{26}$

$r = 6.164414\ units, q = 6.480741 units, p = 5.099020\ units$

$$s = \frac{1}{2}(6.164414 + 6.480741 + 5.099020)$$

$s = 8.872087\ units$

$Area = \sqrt{(8.872087)(3.773067)(2.391346)(2.707673)}$

$Area = 14.7224\ sq.units$

17.a) $p = -\frac{4}{5}$

b i) $r = -3i + j + 5k + \lambda(7i - j - k)$

18.a) 6

b) $r = -4i + 6j - 6k + \lambda(-2i - 14j + 2k)$

c) $-5i - j - 5k$

d) 50.6°

19.a) $r = |\boldsymbol{PQ}| = \sqrt{\{(2+3)^2 + (-1-2)^2 + (3-1)^2\}}$ $ALT: \overrightarrow{PQ} = \begin{pmatrix} -5 \\ 3 \\ -2 \end{pmatrix}$

$q = |\boldsymbol{PR}| = \sqrt{\{(2-1)^2 + (-1-3)^2 + (3+2)^2\}}$ $\overrightarrow{PR} = \begin{pmatrix} -1 \\ 4 \\ -5 \end{pmatrix}$. Get normal etc

$p = |\boldsymbol{QR}| = \sqrt{\{(-3-1)^2 + (2-3)^2 + (1+2)^2\}}$

$r = \sqrt{38}$, $q = \sqrt{42}$, $p = \sqrt{26}$

$r = 6.164414\ units, q = 6.480741 units, p = 5.099020\ units$

$$s = \frac{1}{2}(6.164414 + 6.480741 + 5.099020)$$

$s = 8.872087\ units$

$Area = \sqrt{(8.872087)(3.773067)(2.391346)(2.707673)}$

$Area = 14.7224\ sq.units$

20.a) $y = mx + c \dots\dots\dots(i)$ $x^2 + y^2 = r^2 \dots\dots.(ii)$ $c^2 = r^2(1+m^2)$

Putting (i) into (ii) $x^2 + (mx+c)^2 = r^2$

$x^2 + m^2x^2 + 2mcx + c^2 - r^2 = 0$

$(2mc)^2 - 4(1+m^2)(c^2 - r^2) = 0$

$4m^2c^2 - 4(1+m^2)(c^2-r^2) = 0$

$m^2c^2 - (c^2 - r^2 + m^2c^2 - m^2r^2) = 0$

$m^2c^2 - c^2 + r^2 - m^2c^2 + m^2r^2 = 0$

$c^2 = r^2(1+m^2)$

b) At $(-1,13)$ $13 = -m + c \dots.(i)$ $With\ r^2 = 10$

$c^2 = 10(1+m^2) \dots\dots\dots\dots\dots\dots(ii)$

$(13+m)^2 = 10(1+m^2)$

$169 + 26m + m^2 = 10 + 10m^2$

$9m^2 - 26m - 159 = 0$

$m = \frac{26 \pm \sqrt{(-26)^2 - 4 \times 9 \times -159}}{18}$

$m = \frac{53}{9}$ $or\ m = -3$

$c = \frac{53}{9} + 13 = \frac{170}{9}$ $or\ c = -3 + 13 = 10$

$y = \frac{53}{9}x + \frac{170}{9}$ $or\ y = -3x + 10$

21.a) At t, p =q =t for the tangent

$t^2y + x = 4t$

b) Tangent $y = -\frac{1}{t^2}x + \frac{4}{t}$ $has\ slope\ m_1 = -\frac{1}{t^2}$

$Normal\ at\ T\ has\ slope = m_2 = t^2$

$\frac{y - \frac{2}{t}}{x - 2t} = t^2$

$y - \frac{2}{t} = t^2(x - 2t)$

$ty - 2 = t^3x - 2t^4$

$ty - t^3x + 2t^4 - 2 = 0$ $is\ the\ normal$

Normal meets curve again, first at $x_1 = 2t, y_1 = \frac{2}{t}$

$xy = 4 \ldots \ldots (i)$ and $ty - t^3x + 2t^4 - 2 = 0 \ldots . (ii)$

$txy - t^3x^2 + (2t^4 - 2)x = 0$

$4t - t^3x^2 + (2t^4 - 2)x = 0$ $with\ roots\ x_1 = 2t,\ x_2 = ?$

$x_1x_2 = \frac{4t}{-t^3}$

$x_2 = \frac{4t}{-t^3} \div 2t$

$x_2 = -\frac{2}{t^3}$

$y_2 = \frac{4}{x_2} = 4 \div \frac{-2}{t^3}$

$y_2 = -2t^3$

$R(-\frac{2}{t^3}, -2t^3)$

22.a) $y = -3x + 25$

b) $(x-7)^2 + (y-4)^2 = \frac{5}{2}$

c) $k = \frac{10}{3}$

23.a)) $f'(x) = 4x + 4 = 0$

$\therefore x = -1$

$f''(x) = 4 < 0, hence\ minimum\ at\ x = -1.$

$f(-1) = 2(-1)^2 + 4(-1) - 6 = -8$

$(-1, -8)\ minimum\ point$

$Intercepts, x = 0\ at\ f(0) = -6, at\ (0, -6)$

$f(x) = 2x^2 + 4x - 6 = 0$

$(x+3)(x-1) = 0$

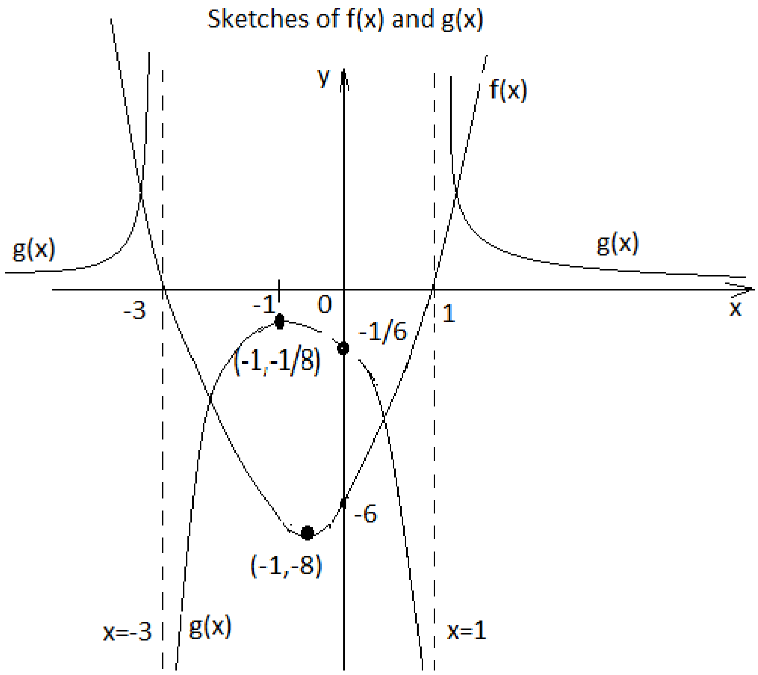
$x = -3\ or\ x = 1$

$(-3,0) and\ (1,0)$

$g(x) has\ maximum\ at\ \left(-1, -\frac{1}{8}\right)$

And vertical asymptotes at $x = -3\ and\ x = 1$

As $x \to \pm\infty, g(x) \to 0$



b) Volume= $\int_{-1}^{0}\pi(2x^2+4x-6)^2\,dx$

$= \int_{-1}^{0}\pi(4x^4+16x^3-8x^2-48x+36)dx\,dx$

$= \pi[\frac{4}{5}x^5+4x^4-\frac{8}{3}x^3-24x^2+36x]_{-1}^{0}$

$= 0-\pi[\frac{4}{5}(-1)^5+4(-1)^4-\frac{8}{3}(-1)^3-24(-1)^2+36(-1)]$

$Volume = \frac{812}{15}\pi$ cubic. units

24.a) a) $tan30^o = \frac{1}{\sqrt{3}} = \frac{r}{h}$

$r = \frac{1}{\sqrt{3}}h, \quad r^2 = \frac{h^2}{3}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi.\frac{h^2}{3}.h = \frac{1}{9}\pi h^3$

$\frac{dS}{dt} = \frac{dS}{dh} \times \frac{dh}{dV} \times \frac{dV}{dt}$ where $S = \pi r^2 = \frac{1}{3}\pi h^2$ and $\frac{dV}{dt} = 9\ cm^3/s$

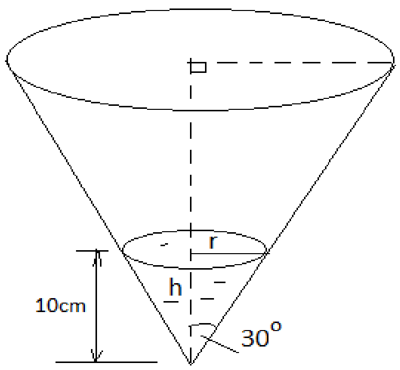
$\frac{dS}{dh} = \frac{2}{3}\pi h, \qquad \frac{dV}{dh} = \frac{1}{3}\pi h^2$

$\frac{dS}{dt} = \frac{2}{3}\pi h \times \frac{3}{\pi h^2} \times 9$

$\frac{dS}{dt} = \frac{18}{h}$

At $h = 10cm,\ \frac{dS}{dt} = \frac{18}{10}$

$\frac{dS}{dt} = 1.8\ cm^2/s$



b) $R^2 = (2r)^2 + (\frac{h}{2})^2$

$R^2 = 4r^2 + \frac{h^2}{4}$

$V = \pi r^2 h$

$V = \frac{\pi}{4}h(R^2 - \frac{h^2}{4})$

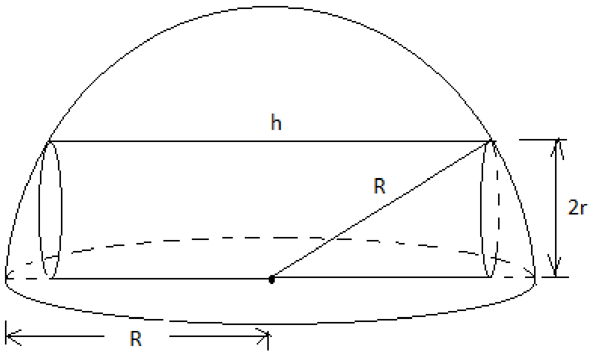
For max or min volume, $\frac{dV}{dh} = \frac{\pi R^2}{4} - \frac{3\pi h^2}{16} = 0$

$R^2 = \frac{3h^2}{4}$ thus $h = \sqrt{\frac{4R^2}{3}} = \frac{2\sqrt{3}\,R}{3}$

$\frac{d^2V}{dh^2} = -\frac{3\pi h}{8} < 0$ $hence$ max $volume$

$V_{max} = \frac{\pi}{4}.\frac{2\sqrt{3}\,R}{3}.(R^2 - \frac{R^2}{3})$

$V_{max} = \frac{\sqrt{3}\,\pi R^3}{9}$



25.a) Let P = population that has heard the rumour at time, t hours after 8:00 a.m.

$\frac{dP}{dt} \propto P(1000 - P)$

$\frac{dP}{dt} = kP(1000 - P)$

$At\ P = 100,\ \frac{dP}{dt} = 10\ people\ per\ hour$

$10 = k(100)(1000 - 100)$

$k = \frac{1}{9000}$

$\Longrightarrow \frac{dP}{dt} = \frac{P}{9000}(1000 - P)\ \ is\ the\ diff.equation.$

b) (i) $\int \frac{dP}{P(1000-P)} = \frac{1}{9000}\int dt$

$\frac{1}{P(1000-P)} = \frac{A}{P} + \frac{B}{1000-P}$

$$1 = A(1000 - P) + BP$$

$Put\ P = 0,\ 1 = 1000A$ thus $A = \frac{1}{1000}$

$Put\ P = 1000,\ 1 = 1000B$ thus $B = \frac{1}{1000}$

$$\frac{1}{1000}\int\frac{dP}{P} + \frac{1}{1000}\int\frac{dP}{(1000-P)} = \frac{1}{9000}\int dt$$

$$lnP - \ln(1000 - P) = \frac{t}{9} + c$$

$At\ t = 0\ hour, P = 100\ people$

$$ln100 - ln900 = c$$
$$c = -ln9$$

$$lnP - \ln(1000 - P) = \frac{t}{9} - ln9$$

$$\ln(\frac{9P}{1000-P}) = \frac{t}{9}$$

$$\frac{9P}{1000-P} = e^{\frac{t}{9}}$$

$$9P = 1000e^{\frac{t}{9}} - Pe^{\frac{t}{9}}$$

$$P\left(9 + e^{\frac{t}{9}}\right) = 1000e^{\frac{t}{9}}$$

$$P = \frac{1000e^{\frac{t}{9}}}{9+e^{\frac{t}{9}}}$$

$At\ t = 24\ hours, P = ?$

$$P = \frac{1000e^{\frac{24}{9}}}{9+e^{\frac{24}{9}}}$$

$$P = 615.25170 = 615\ people(0\ d.p)$$

b) (ii) $At\ P = 900\ people, t = ?$

$$t = 9\ln(\frac{9P}{1000-P})$$

$$t = 9\ln(\frac{9\times900}{1000-900})$$

$$t = 39.5500\ hours = 39\ hours\ 33\ minutes$$

Required time $= 8:00 a.m + 39\ hours\ 33\ minutes$

$= 11:33\ p.m\ on\ the\ next\ day.$

26.a) $cos6y = 2cos^2 3y - 1 \Rightarrow \int cos^2 3ysinydy = \frac{1}{2}\int(1 + cos6y)sinydy$

$\therefore \int cos^2 3ysinydy = \frac{1}{2}\int sinydy + \frac{1}{2}\int cos6ysinydy$

$= \frac{1}{2}\int sinydy + \frac{1}{4}\int 2cos6ysinydy$

$= \frac{1}{2}\int sinydy + \frac{1}{4}\int(sin6y - sin5y)dy$

$= -\frac{1}{2}cosy + \frac{1}{28}cos7y - \frac{1}{20}cos5y + c$

b) $I = \int_0^{1/4}\sqrt{\frac{1-2x}{1+2x}}$ dx $= \int_0^{1/4}\sqrt{\frac{(1-2x)^2}{1-4x^2}}$ dx

$dx = \frac{1}{2}cosudu$

x	u
¼	$\pi/6$
0	0

$I = \int_0^{\pi/6}\frac{1-sinu}{\sqrt{1-sin^2u}}.\frac{1}{2}cosudu$

$I = \int_0^{\pi/6}(1 - sinu)du$

$= \frac{1}{2}[u + cosu]_0^{\frac{\pi}{6}}$

$= \frac{1}{2}\left[\frac{\pi}{6} + \frac{cos\pi}{6}\right] - \frac{1}{2}cos0$

$= \frac{\pi}{12} + \frac{\sqrt{3}}{4} - \frac{1}{2}$

MECHANICS

NUMBER 1

a)

i) $v(t) = \frac{d}{dt}[r(t)]$

$= \frac{d}{dt}[4 - 2\sin 2t]$

$= -4\cos 2t$

ii) $a = \frac{d}{dt}[v(t)]$

$= \frac{d}{dt}[-4cos2t]$

$= 8\sin 2t$

b)

$i)\ initial\ displacement$

$r(0) = 4 - 2\sin 0$

$= 4$

$ii)\ initial\ velocity$

$v(0) = -4\cos 0$

$= -4$

$iii)\ initial\ acceleration$

$a = 8\sin 0$

$= 0$

c) the time when the particle is

$i)\ at\ rest$

$v(t) = 0$

$-\cos 2t = 0$

$2t = \cos^{-1} 0$

$= \frac{\pi}{2}, \frac{3\pi}{2}$

$t = \frac{\pi}{4}, \frac{3\pi}{4}$

The critical points give us the intervals

$$\left[0, \frac{\pi}{4}\right], \quad \left[\frac{\pi}{4}, \frac{3\pi}{4}\right], \left[\frac{3\pi}{4}, \pi\right]$$

$$v\left(\frac{\pi}{8}\right) = -4\cos\frac{\pi}{8}$$

$$= -ve < 0\{moving\ to\ the\ left\}$$

$$v\left(\frac{\pi}{2}\right) = -4\cos 2\left(\frac{\pi}{2}\right)$$

$$= +ve \ > 0\{moving\ to\ the\ right\}$$

$$v\left(\frac{7\pi}{8}\right) = -4\cos 2\left(\frac{7\pi}{8}\right)$$

$$= -ve \ < 0\{moving\ to\ the\ left\}$$

NUMBER 2

$$let\ m\ be\ the\ mass\ per\ unit\ area$$

SHAPE	AREA	WEIGHT	DISTANCE OF CG FROM BASE	MOMENT ABOUT THE BASE
TRIANGLE	$0.4y$	$0.4ymg$	$\frac{1}{2}y$	$0.2y^2mg$
RECTANGLE	y^2	y^2mg	$\frac{2}{3}y$	$\frac{2}{3}y^3mg$
COMPOSITE		$(0.4y + y^2)mg$	$\bar{x}$	$(0.4y + y^2)mg\bar{x}$

$$(0.4y + y^2)mg\bar{x} = \frac{2}{3}y^3mg + 0.2y^2mg$$

$$\bar{x} = 0.171m\ (\,when\ y\ = 0.3)$$

b)

$$for\ toppling\ to\ occur:$$

$$the\ moment\ of\ the\ rectangle = \ the\ moment\ of\ the\ traingle$$

$$at\ the\ interface$$

$$0.4ymg \times 0.2 = y^2mg \times \frac{2}{3}y$$

$$y = 0.346m$$

NUMBER 3

$$(i)$$

$$a = \frac{F}{m}$$

$$= \frac{14i + 21j + 24k}{4}$$

$$= \tfrac{7}{2} i + \tfrac{21}{4} j + 6k$$

$$(ii)$$

$$v = \int a \, dt$$

$$= \int \frac{7}{2} i + \frac{21}{4} j + 6k \, dt$$

$$v = \frac{7}{2} t \, i + \frac{21}{4} t \, j + 6t \, k + c$$

$$when\ t = 0, v = 0 \ \ there\ fore;\ c = 0$$

$$v = \frac{7}{2} t \, i + \frac{21}{4} t \, j + 6t \, k$$

$$v(2) = \frac{7}{2} (2) \, i + \frac{21}{4} (2) \, j + 6(2) \, k$$

$$= 7i + 10.5j + 12k$$

Speed = |v(2)|

=17.414m/s

$$displacement, r(t) = \int v \, dt$$

$$= \int \frac{7}{2} t \, i + \frac{21}{4} t \, j + 6t \, k \, dt$$

$$r(t) = \frac{7}{4} t^2 \, i + \frac{21}{8} t^2 \, j + 3t^2 + c$$

$$when\ t = 0, r = -4i + 2j$$

$$C = \ -4i + 2j$$

$$r(t) = \left(\frac{7}{4} t^2 - 4\right) i + (\frac{21}{8} t^2 + 2) \, j + 3t^2 k$$

$$r(4) = (\frac{7}{4} (4)^2 - 4) \, i + (\frac{21}{8} (4)^2 + 2) \, j + 3(4)^2$$

$= 24\, i + 44\, j + 48\, k$

Distance = |r(4)|

= 69.3974m

NUMBER 4

a) $Time\ to\ reach\ maximum\ height$

$$v = u + at$$

$$T = \frac{u \sin\theta}{g}$$

$$= \frac{60 \sin 30}{10}$$

$$= 3\ s$$

$b)\ the\ maximum\ height$

$$v^2 = u^2 + 2as$$

$$0 = (u \sin\theta)^2 - 2gH$$

$$H = \frac{(60 \sin 30)^2}{2(10)}$$

$$= 45\ m$$

$c)\ time\ of\ flight$

$$T = 2(3)\ \ s$$

$d)\ the\ horizontal\ range\ of\ the\ particle$

$$x = u\ cos\,\theta\ t$$

$$= 60 \cos 30\ \ (6)$$

$$= 311.77m$$

NUMBER 5

Initially

$$k.e = \frac{1}{2}(2.8)(6)^2$$
$$p.e = 2.8(9.8)(0.75)$$

$After\ making\ an\ angle\boldsymbol{\theta}$

$$k.e = \frac{1}{2}(2.8)v^2$$

$$p.e = 2.8(9.8)(0.75)\cos\theta$$

$$total\ energy\ before = total\ energy\ after$$

$$ke_{before} + p.e_{before} = ke_{after} + pe_{after}$$

$$50.4 + 20.58 = 1.4v^2 + 20.58\cos\theta$$

$$\therefore v^2 = 50.7 - 14.7\cos\theta$$

$$b)\ for\ circular\ motion\ \ T = m\frac{v^2}{r} + mg\cos\theta$$

$$200 = 2.8\left(\frac{50.7 - 14.7\cos\theta}{0.75}\right) + 2.8(9.8)\cos\theta$$

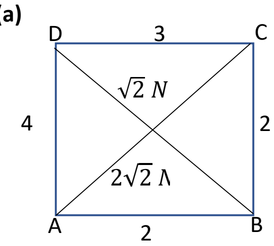
$$200 \times 0.75 = 2.8\left(\frac{50.7 - 14.7\cos\theta}{0.75}\right) \times 0.75 + 2.8(9.8)\cos\theta \times 0.75$$

$$\cos\theta = -\ 0.3907$$

$$\theta = 113^o$$

Number 6

(a)



$$\boldsymbol{F_X = 2 + 3 + 2\sqrt{2}COS\ 45 + \sqrt{2}COS\ 45}$$

$$\boldsymbol{F_X = 8N}$$

$$\boldsymbol{F_Y = 2 + 4 + 2\sqrt{2}sin\ 45 - \sqrt{2}sin\ 45}$$

$$\boldsymbol{F_y = 7N}$$

$$\boldsymbol{Resultant\ = \sqrt{8^2 + 7^2}}$$

= 10.63N

(b)

$$\boldsymbol{xF_X - yF_x = m}$$

$$\boldsymbol{7x - 8y = -4}$$

NUMBER 7

$a)$ *position of B relative to A at any time t*

$$r_A(t) = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -6 \\ 0 \\ 1 \end{pmatrix} t$$

$$r_B(t) = \begin{pmatrix} 4 \\ -14 \\ 1 \end{pmatrix} + \begin{pmatrix} -5 \\ 1 \\ 7 \end{pmatrix} t$$

$$_Br_A\,(t) = r_B(t) - r_A(t)$$

$$\left[\begin{pmatrix} 4 \\ -14 \\ 1 \end{pmatrix} + \begin{pmatrix} -5 \\ 1 \\ 7 \end{pmatrix} t\right] - \left[\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -6 \\ 0 \\ 1 \end{pmatrix} t\right]$$

$$= \begin{pmatrix} 3 + t \\ -16 + t \\ -2 + 6t \end{pmatrix}$$

$b)$ *time that elapses before the trains are closest to each other*

$$\left|\,_Br_A\,(t)\right|^2 = (3 + t)^2 + (-16 + t)^2 + (-2 + 6t)^2$$

$$\left|\,_Br_A\,(t)\right|^2 = 9 + 6t + t^2 + 256 - 32t + t^2 + 4 - 24t + 36t^2$$

$$\left|\,_Br_A\,(t)\right|^2 = 38t^2 - 50t + 269$$

$$\frac{d}{dt}\left[\left|\,_Br_A\,(t)\right|^2\right] = 76t - 50$$

$$76t - 50 = 0$$

$$t = \frac{25}{38}\,s$$

$c)$ *least distance between the trains in the subsequent motion*

$$\left|\,_Br_A\left(\frac{25}{38}\right)\right|^2 = 38\left(\frac{25}{38}\right)^2 - 50\left(\frac{25}{38}\right) + 269$$

$$d = \sqrt{252.5526}$$

$$= 15.891\,m$$

NUMBER 9

$$F - R_F = ma$$

$$\frac{28000}{v} - 4v = 1000a$$

$$a = \frac{28000 - 4v^2}{1000v}$$

$$= \frac{7000 - v^2}{250v}$$

$$\frac{dv}{dt} = \frac{7000 - v^2}{250v}$$

$b)\ the\ time\ taken\ for\ the\ car\ to\ pass\ between\ A\ and\ B$

$$\frac{250v}{7000 - v^2} dv = dt$$

$Intergrating\ either\ side$

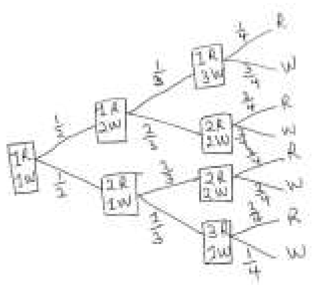
$$\int_{10}^{40} \frac{250v}{7000 - v^2} dv = \int dt$$

$$t = -125In[7000 - v^2]_{10}^{40}$$

$$= 30.64\ s$$

	PROBABILITY SOLUTIONS
10	$E(X) = \int_{-\infty}^{\infty} xf(x)dx$

a,	$$= \int_{-3}^{3} x\left(\frac{1(9-x^2)}{36}\right)dx$$ $$E(X) = 0$$ $$Var(X) = E(X^2) - E(X)$$ $$= \int_{-1}^{1} x^2\left(\frac{1(9-x^2)}{36}\right)dx - 0$$ $$Var(X) = 1.8$$
b,I	$$P(X > 2) = \int_{2}^{3}\left(\frac{1(9-x^2)}{36}\right)dx$$ $$= \frac{2}{27}$$
II	$$\sigma = \sqrt{Variance} = \sqrt{1.8} = 1.3416$$ $$P(\lvert X\rvert > \sigma) = P(X < -\sigma) + P(X > \sigma)$$ $$P(\lvert X\rvert > 1.3416) = P(X < -1.3416) + P(X > 1.3416)$$ $$= \int_{-3}^{-1.3416} \frac{1(9-x^2)}{36}dx + \int_{1.3416}^{3} \frac{1(9-x^2)}{36}dx$$ $$= 0.186959 + 0.186959$$ $$\approx 0.3739$$
11 a	$$\int_{-\infty}^{\infty} f(x)dx = 1$$ $$\int_{0}^{24} \frac{(x-18)^2}{k}dx = 1$$ $$\frac{(24-18)^3 - (0-18)^3}{3k} = 1$$ $$\frac{2016}{k} = 1$$ $$k = 2016$$
b,	$$P(X < 2) = \int_{0}^{2} \frac{(x-18)^2}{2016}dx$$ $$\frac{(2-18)^3 - (0-18)^3}{3 \times 2016}$$ $$P(X < 2) = \frac{31}{108}$$ $$\frac{31}{108} \times 365 = 104.7685$$ $$\approx 105\ Days$$
c,	$$E(X) = \int_{-\infty}^{\infty} xf(x)dx$$ $$= \int_{0}^{24} x\frac{(x-18)^2}{2016}dx$$ $$\frac{1}{2016}\int_{0}^{24}(x^3 - 36x^2 + 324x)dx$$ $$\frac{10368}{2016}$$ $$E(X) = 5.1429$$

12 a	$F(1.5) = 1$ $k(2(1.5)^3 - 1.5^4) = 1$ $\frac{27}{16}k = 1$ $k = \frac{16}{27}$
b,	P(Students who spent more than 1 hour on homework)=$P(t > 1)$ $P(t > 1) = F(1.5) - F(1)$ $1 - \frac{16}{27}(2(1)^3 - 1^4)$ $P(t > 1) = \frac{11}{27}$
c,	$F(t) = \begin{cases} 0, & t < 0 \\ k(2t^3 - t^4), & 0 \le t \le 1.5 \\ 1, & t > 1.5 \end{cases}$ $f(t) = \frac{d}{dt}(F(t)$ $t < 0, f(t) = \frac{d}{dt}(0) = 0$ $0 \le t \le 1.5, f(t) = \frac{d}{dt}\left(\frac{16}{27}(2t^3 - t^4)\right) = \frac{16}{27}(6t^2 - 4t^3)$ $t > 1.5, f(t) = \frac{d}{dt}(1) = 0$ $f(t) = \begin{cases} \frac{16}{27}(6t^2 - 4t^3), 0 \le t \le 1.5 \\ 0, \quad otherwise \end{cases}$
d,	$E(T) = \int_{-\infty}^{\infty} tf(t)dt$ $= \int_0^{1.5} t\frac{16}{27}(6t^2 - 4t^3)dt$ $E(T) = 0.9$
13	 $P(RnRnR) = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} = \frac{1}{24}$ $P(RnRnW) = \frac{1}{2} \times \frac{1}{3} \times \frac{3}{4} = \frac{1}{8}$

$$P(RnWnR) = \frac{1}{2} \times \frac{2}{3} \times \frac{2}{4} = \frac{1}{6}$$
$$P(RnWnW) = \frac{1}{2} \times \frac{2}{3} \times \frac{2}{4} = \frac{1}{6}$$
$$P(WnRnR) = \frac{1}{2} \times \frac{2}{3} \times \frac{2}{4} = \frac{1}{6}$$
$$P(WnRnW) = \frac{1}{2} \times \frac{2}{3} \times \frac{2}{4} = \frac{1}{6}$$
$$P(WnWnR) = \frac{1}{2} \times \frac{1}{3} \times \frac{3}{4} = \frac{1}{8}$$
$$P(WnWnW) = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} = \frac{1}{24}$$

a

$$P(Second\ and\ third\ bead\ are\ the\ same\ colour)$$
$$= P(RnRnR) + P(RnWnW) + P(WnRnR) + P(WnWnW)$$
$$= \frac{1}{24} + \frac{1}{6} + \frac{1}{6} + \frac{1}{24} = \frac{5}{12}$$

b (i)

Let x be the number of red sweets

$$P(X = 0) = P(WnWnW) = \frac{1}{24}$$
$$P(X = 1) = P(RnWnW) + P(WnRnW) + P(WnWnR) = \frac{1}{6} + \frac{1}{6} + \frac{1}{8} = \frac{11}{24}$$
$$P(X = 2) = P(RnRnW) + P(RnWnR) + P(WnRnR) = \frac{1}{8} + \frac{1}{6} + \frac{1}{6} = \frac{11}{24}$$
$$P(X = 3) = P(RnRnR) = \frac{1}{24}$$

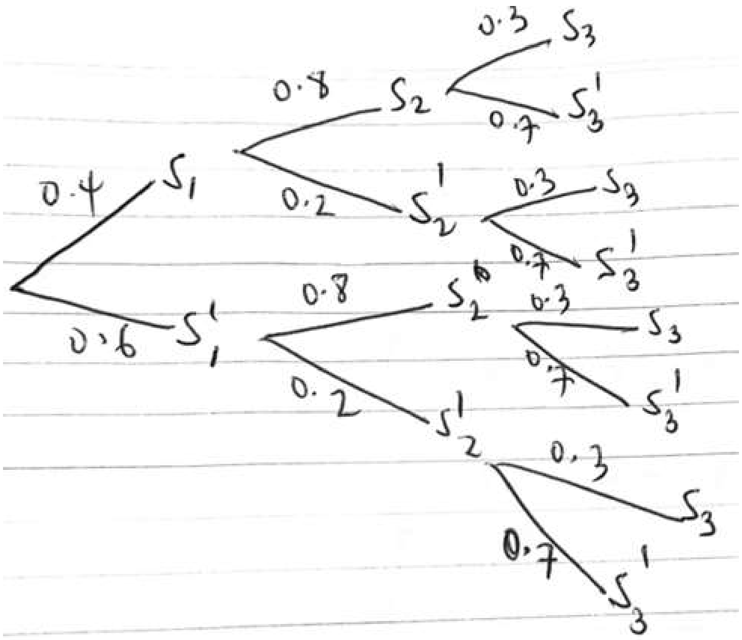
x	$P(X = x)$	$xP(X = x)$
0	$\frac{1}{24}$	0
1	$\frac{11}{24}$	$\frac{11}{24}$
2	$\frac{11}{24}$	$\frac{22}{24}$
3	$\frac{1}{24}$	$\frac{3}{24}$
	$\sum xP(X = x) = \frac{36}{24} = 1.5$	

Expected number of red sweets in the box=1.5

14 (a) 0.813

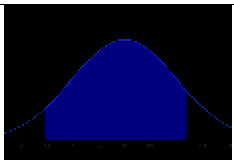
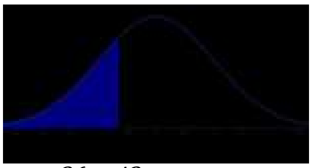
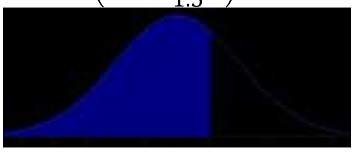
(b) 0.556

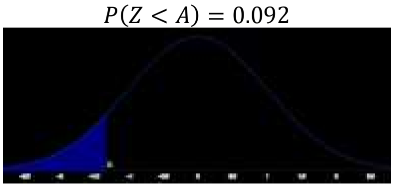
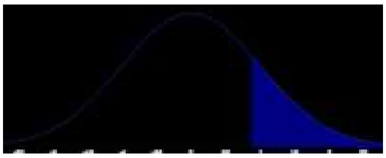
15 a	$$a = P(1head) = 0.7 \times 0.5^3 + 0.3 \times 0.5^3 \times 3$$ $$= \frac{1}{5}$$ $$b = 0.7 \times 0.5^3 \times 3 + 0.3 \times 0.5^3 \times 3$$ $$= \frac{3}{8}$$ $$c = 0.7 \times 0.5^3 \times 3 + 0.3 \times 0.5^3$$ $$= \frac{3}{10}$$
b	$$E(X) = \sum xP(X = x)$$ $$E(X) = 0 \times \frac{3}{80} + 1 \times \frac{1}{5} + 2 \times \frac{3}{8} + 3 \times \frac{3}{10} + 4 \times \frac{7}{80}$$ $$E(X) = \frac{176}{80}$$ $$= 2.2$$
16 I	Let the probability that Masandugu has to stop at the first second and third sets of lights be P(S1),P(S2) and P(S3) respectively.

	
II	probability that Masandugu has to stop at each of the first two sets of lights but does not have to stop at the third set= $P(S1nS2nS'3)$ $$P(S1nS2nS'3) = P(S1) \times P(S2) \times P(S'3)$$ $$= 0.4 \times 0.8 \times 0.7$$ $$= 0.224$$
III	Probability that Masandugu has to stop at exactly two of the three sets of lights $$= P(S1nS2nS'3) + P(S1nS'2nS3) + P(S'1nS2nS3)$$ $$= 0.4 \times 0.8 \times 0.7 + 0.4 \times 0.2 \times 0.3 + 0.6 \times 0.8 \times 0.3$$ $$= 0.392$$
IV	probability that Masandugu has to stop at the first set of lights, given that she has to stop at exactly two sets of lights= $P(S1/\text{ Masandugu has to stop at each of the first two sets})$ $$= \frac{P(S1nS2nS'3) + P(S1nS'2nS3)}{P(S1nS2nS'3) + P(S1nS'2nS3) + P(S'1nS2nS3)}$$ $$= \frac{0.248}{0.392}$$ $$\approx 0.6327$$
17 a	Let leopards be X $$X \sim N(55, 6^2)$$ $$P(46 < X < 62) = P(\frac{46-55}{6} < Z < \frac{62-55}{6})$$ $$= P(-1.5 < Z < 1.167)$$

Commented [NC1]:

Commented [NC2]:

	 $= \phi(1.167) + \phi(-1.5)$ $= 0.37839 + 0.43319$ $= 0.81158(Calc)$
B	$l \sim N(42, \sigma^2)$ $P(l < 36) = 25\%$ $P\left(Z < \frac{36-42}{\sigma}\right) = 0.25$ $Let\ a = \frac{36-42}{\sigma}$ $P(Z < a) = 0.5$  $\frac{36-42}{\sigma} = -0.674$ $\sigma = 8.9\ kg$
18 a	Let the length be x $X \sim N(5.2, 1.5^2)$ $P(X < 6) = P\left(Z < \frac{6-5.2}{1.5}\right) = P(Z < 0.533)$  $= 0.5 + \phi(0.533)$ $= 0.5 + 0.2031$ $= 0.7031$
b	$X \sim N(\mu, \sigma^2)$ $P(X < 3) = \frac{46}{500} = 0.092$ $P(X > 8) = \frac{95}{500} = 0.19$ $P\left(Z < \frac{3-\mu}{\sigma}\right) = 0.092$ $let\ A = \frac{3-\mu}{\sigma}$

	$$P(Z < A) = 0.092$$  $$-A = 1.33$$ $$3 - \mu = 1.33\sigma \ldots\ldots\ldots\ldots\ldots\ldots.eqn\ 1$$ $$P\left(Z > \frac{8-\mu}{\sigma}\right) = 0.19$$ $$let\ B = \frac{8-\mu}{\sigma}$$ $$P(Z > B) = 0.19$$  $$B = 0.88$$ $$8 - \mu = 0.88\sigma \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.eqn\ 2$$ Solving eqn 1 and eqn 2 $$3 - \mu = 1.33\sigma$$ $$8 - \mu = 0.88\sigma$$ $$\sigma = \frac{5}{2.21} = 2.26$$ $$\mu = 3 + 1.33 \times 2.26 = 6.01$$

STATISTICS

19 a	

Weights	f	Class boundaries	x	xf	X^2f	F	$i\ or\ cw$	f.d
20-24	3	19.5-24.5	22	66	1452	3	5	0.6
25-29	5	24.5-29.5	27	135	3646	8	5	1
30	2	29.5-30.5	30	60	1800	10	1	2
31-34	6	30.5-34.5	32.5	195	6337.5	16	4	1.5
35-49	9	34.5-49.5	42	378	15876	25	15	0.6
sum				834	29110.5			

	Mean weight $=\frac{834}{25}$ $= 33.36kg$ Standard deviation $=\sqrt{\left(\frac{29110.5}{25}\right) - (\frac{834}{25})^2}$ $= 7.1785kg$
ii)	(tables below)

Weight	24.5	26.5	29.5
F	3	x	8

$\frac{x-3}{26.5-24.5} = \frac{8-3}{29.5-24.5}$

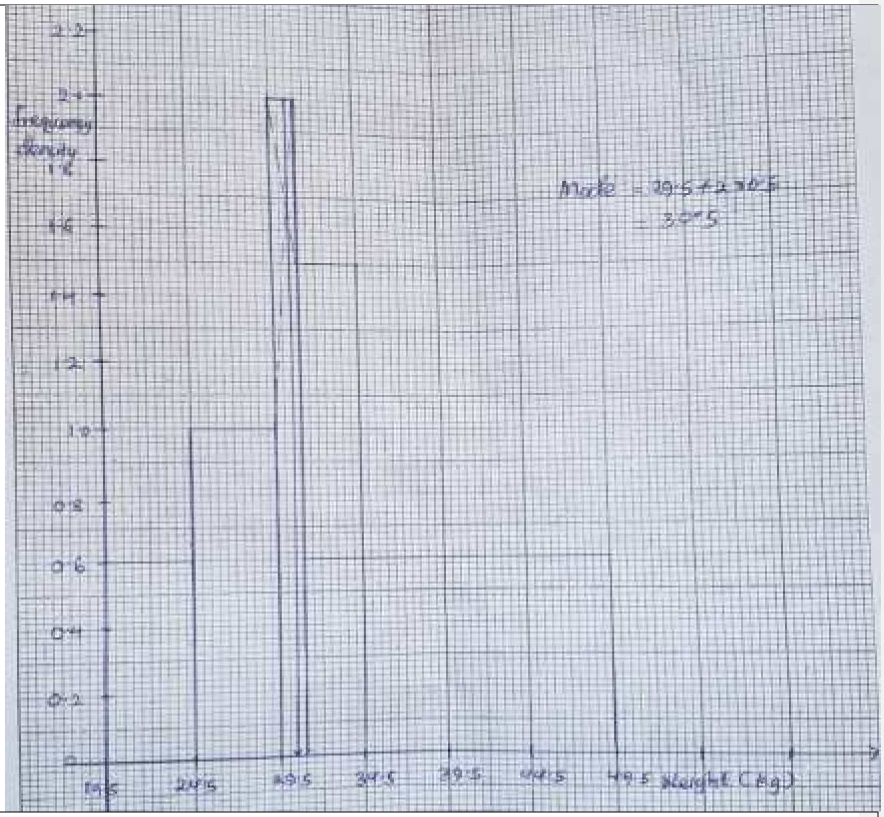
X=5

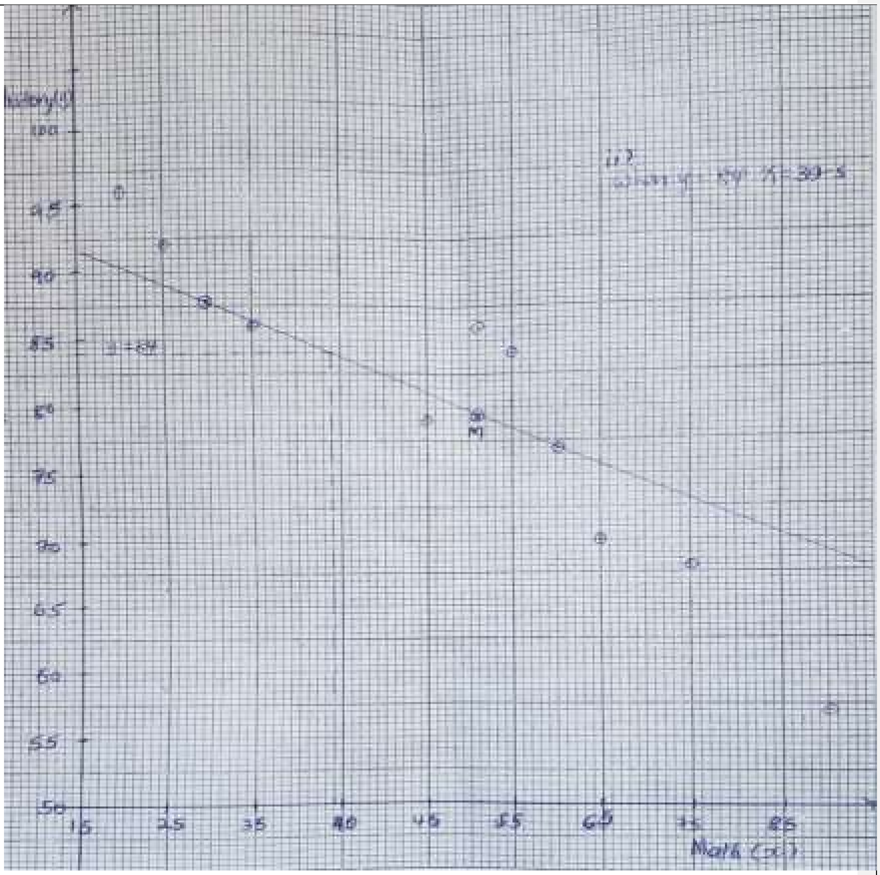
Weight	30.5	32.5	34.5
F	10	a	16

$\frac{a-10}{32.5-30.5} = \frac{16-10}{34.5-30.5}$

a=13

$$\therefore between = 13 - 5$$
$$= 8boys$$

B	2·2 2·0 Frequency density 1·8 1·6 1·4 1·2 1·0 0·8 0·6 0·4 0·2 0 Mode = 29·5 + 2 × 0·5 = 30·5 19·5 24·5 29·5 34·5 39·5 44·5 49·5 Weight (Kg)
20 a	i)



iii)

X	y	Rx	Ry	$\lvert d \rvert$	d^2
35	86	3	7.5	4.5	20.25
65	70	8	3	5	25
55	84	6	6	0	0
25	92	2	9	7	49
45	79	4	5	1	1
75	68	9	2	7	49
20	96	1	10	9	81
90	58	10	1	9	81
51	86	5	7.5	2.5	6.25
60	77	7	4	3	9

$$\rho = 1 - \frac{6x272.5}{10(99)}$$

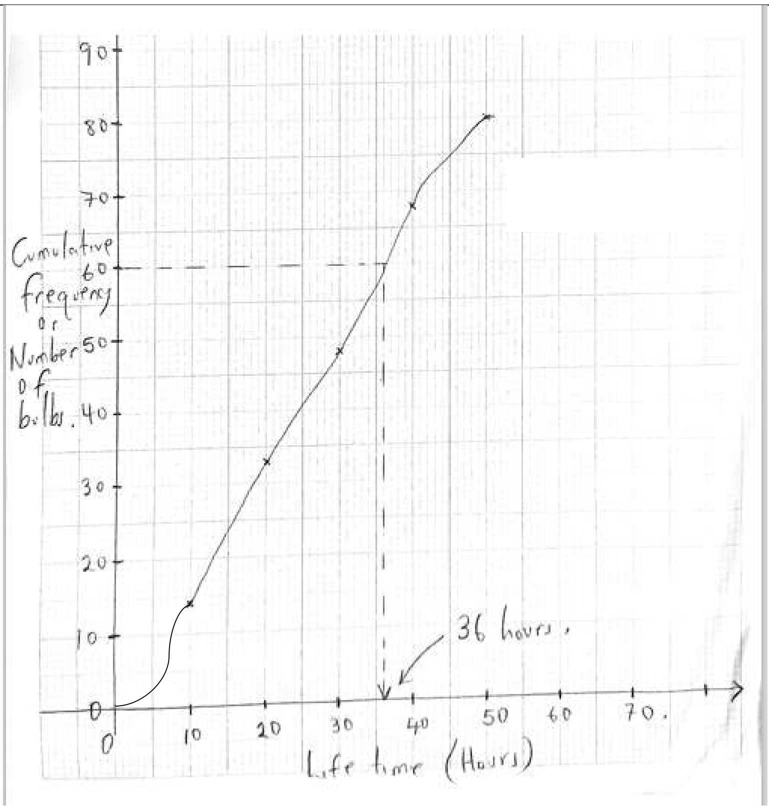
$$= -0.6515$$

It is significant at 5% since |0.6515| exceeds 0.65 .

21 a

Class	f	x	xf	X^2f	Cf
0-10	14	5	70	350	14
10-20	19	15	285	4275	33
20-30	15	25	375	9375	48
30-40	20	35	700	24500	68
40-50	12	45	540	24300	80
	$\sum f = 80$		$\sum fx =$ 1970	$\sum fx^2 =$ 62800	

$$Mean = \frac{1970}{80}$$
$$Mean = 24.625$$
$$Variance = \frac{62800}{80} - 24.625^2$$
$$= 178.6094$$

b	 Exceeded by 75% = 50 -36 = 14 hours.
22 a	Price index $=\frac{price\ of\ the\ current\ year}{price\ of\ the\ base}$ For X, $125=\frac{P}{3200}\times 100$ Price of X in 2024,P=4000 For Y, 105=$\frac{P}{4000}$ Price of Y in 2024,P= 4200 For Z, $120=\frac{P}{4500}$ Price of Z,P=5400
b	Composite price index= $\frac{\sum IW}{\sum W}$ I-price index W-weights

	Composite price index= $\frac{3\times125+5\times105+2\times120}{3+5+2}$ Composite price index= 114
c	Weighted aggregate price index= $\frac{\sum PiW}{\sum PoW}\times 100$ $=\frac{3\times4000+5\times4200+2\times5400}{3\times3200+5\times4000+2\times4500}\times 100$ $=113.47$
23	
(a)	P(A)= 0.5
(b)	$p(A^1/B^1)=0.25$
(c)	0.7

NUMERICAL ANALYSIS

NUMBER 24

$$h=\frac{b-a}{n-1}$$

$$h=\frac{6-0}{7-1}$$

$$=1$$

x	xe^{-x}	
0	0	
1		0.3679
2		0.2707
3		0.1494
4		0.0733
5		0.0337
6	0.0149	
	0.0149	**1.1983**

$$\int_0^6 xe^{-x}dx \approx \frac{1}{2}[0.0149+2(1.1983)]$$

$$=1.21$$

$b)$

$$\int xe^{-x}dx$$

Using by parts

$$u = x \quad \frac{dv}{dx} = e^{-x}$$

$$\frac{du}{dx} = 1 \quad v = -e^{-x}$$

$$= -xe^{-x} + \int e^{-x}dx$$

$$= [-(x+1)e^{-x}]_0^6$$

$$= 0.98$$

$$Percentage\ error = \frac{1.208 - 0.98}{0.98} \times 100$$

$$= 23.3\%$$

$$-this\ error\ can\ be\ reduced\ by\ increasing\ the\ number\ of\ ordinates.$$

NUMBER 25

77	**a**	**78**
4.3315	**4.6500**	**4.7046**

$$\frac{\mathbf{78 - 77}}{\mathbf{4.7046 - 4.3315}} = \frac{\boldsymbol{a} - \mathbf{77}}{\mathbf{4.65 - 4.3315}}$$

$$a - 77 = \frac{-0.3185}{0.3731}$$

$$a = 77.85$$

$$\therefore \tan^{-1} 4.6500 = 77.85^o$$

78	79	79.6
4.7046	5.1446	b

$$\frac{79.6 - 78}{b - 4.7046} = \frac{79 - 78}{5.1446 - 4.7046}$$

$$b - 4.7046 = 1.6(0.44)$$

$$b = 5.4086$$

$$\therefore \mathbf{\tan 79^o 36' = 5.4086}$$

NUMBER 26

$$x = N^{\frac{1}{6}}$$

$$x^6 = N$$

$$x^6 - N = 0$$

$$f(x) = x^6 - N$$

$$f'(x) = 6x^5$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{({x_n}^6 - N)}{6{x_n}^5}$$

$$= \frac{6{x_n}^6 - ({x_n}^6 - N)}{6{x_n}^5}$$

$$= \frac{5{x_n}^6 + N}{6{x_n}^5}$$

$$x_{n+1} = \frac{5}{6}\left[x_n + \frac{N}{5{x_n}^5}\right]$$

$c)$ *Dry run*

n	x_n	x_{n+1}	$Is\ \lvert x_{n+1} - x_n \rvert \leq 0.0005$
0	1.2	3.0429	NO
1	3.0429	2.5552	NO
2	2.5552	2.1760	NO
…	…	….	…
8		1.7676	YES

The root is 1.768

NUMBER 27

maximum possible error in M

$$(\ error = \frac{1}{2} \times 10^{-n})$$

$$= 0.005$$

maximum possible error in N

$$= 0.0005$$

NUMBER 28

$$Volume\ of\ a\ cone\ , V = \frac{1}{3}\pi r^2 h$$

$$V + \Delta V = \frac{1}{3}\pi(r + \Delta r)^2(h + \Delta h)$$

$$\Delta V = \frac{1}{3}\pi(r + \Delta r)^2(h + \Delta h) - V$$

$$\Delta V = \frac{1}{3}\pi[2rh\Delta h + (\Delta r)^2\, h + r^2\Delta h + 2r\Delta r\Delta h + (\Delta r)^2\Delta h]$$

$$since\ \Delta r <<< r\ and\ \Delta h <<< h,\ \ (\Delta r)^2 \approx 0\ and\ \Delta h\Delta r \approx 0$$

$$\Delta V = \frac{1}{3}\pi[2rh\Delta h + r^2\Delta h]$$

$$\left|\frac{\Delta V}{V}\right| = \left|\frac{\frac{1}{3}\pi[2rh\Delta h + r^2\Delta h]}{\frac{1}{3}\pi[r^2 h]}\right|$$

$$\leq \left|\frac{2\Delta r}{r}\right| + \left|\frac{\Delta h}{h}\right|\ (triangular\ inequality)$$

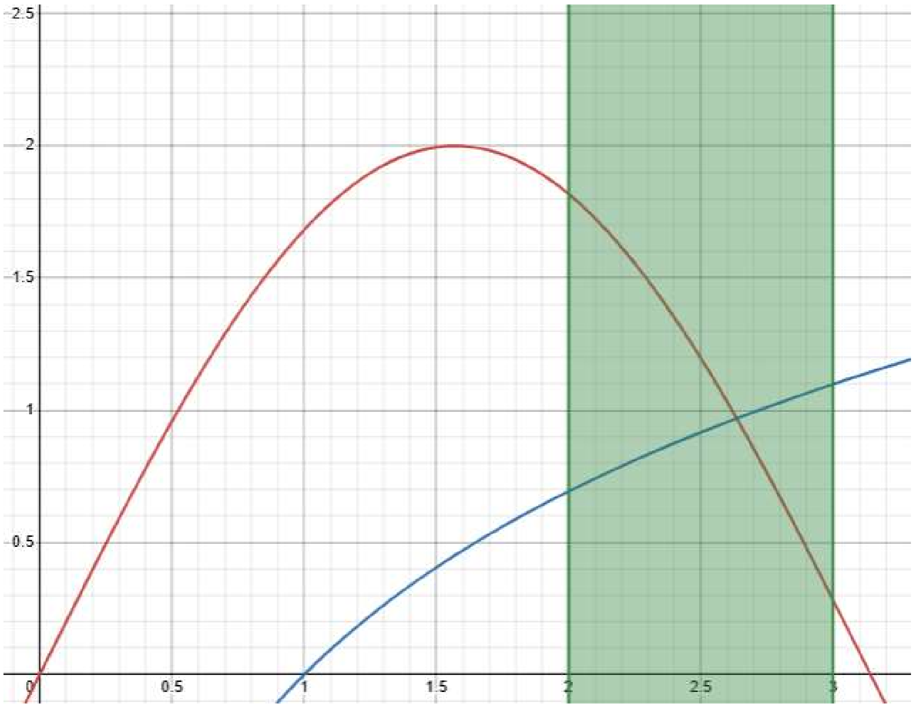
$$= 2\left|\frac{\Delta r}{r}\right| + \left|\frac{\Delta h}{h}\right|$$

$$b)\ percentage\ error$$

$$\left(2\left|\frac{0.005}{3.55}\right| + \left|\frac{0.05}{12.4}\right|\right) \times 100$$

$$= 0.685$$

NUMBER 29



$$f(x) = 2\sin x - Inx$$

$$f'(x) = 2\cos x - \frac{1}{x}$$

$$x_{n+1} = x_n - \frac{2\sin x - Inx}{2\cos x - \frac{1}{x}}$$

$$x_0 = 2.65$$

$$x_1 = (2.65) - \frac{2\sin 2.65 - In2.65}{2\cos 2.65 - \frac{1}{2.65}}$$

$$(make\ sure\ your\ calculator\ is\ in\ rad)$$

$$x_1 = 2.6358$$

$$x_2 = 2.6357$$

$$the\ root\ is\ 2.636$$

NUMBER 30

$Weight\ (kg)$	0.5	1	1.5	2
$cost$	1000	2000	3500	4000

$a)\ let\ the\ charge\ for\ a\ parcel\ of\ 450g\ be\ a$

0.45	0.5	1
a	1000	2000

$$\frac{1-0.5}{2000-1000}=\frac{1-0.45}{2000-a}$$

$$a=900$$

$the\ charge\ for\ 450g\ is\ 900/=$

$let\ the\ charge\ for\ 1.8\ kg\ be\ b$

1.5	1.8	2
3500	b	4000

$$\frac{2-1.5}{4000-3500}=\frac{2-1.8}{4000-b}$$

$$b=3800$$

$The\ charge\ for\ 1.8kg\ is\ 3800/=$

$c)let\ the\ weight\ be\ x$

1.5	2	x
3500	4000	6200

$$\frac{x-1.5}{6200-3500}=\frac{2-1.5}{4000-3500}$$

$$x=4.2\ kg$$

$the\ weight\ of\ the\ parcel\ is\ 4.2\ kg$